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$$+\cot^{-1}(a/d)]-\tfrac{1}{4}b^2[\cot^{-1}(b/c)+\cot^{-1}(b/d)]+\tfrac{1}{4}c^2[\tan^{-1}(a/c)-\tan^{-1}(b/c)]$$

$$+\tfrac{1}{4}d^2[\tan^{-1}(a/d)-\tan^{-1}(b/d)]-\tfrac{1}{4}a(c+d)+\tfrac{1}{4}b(c+d)$$

$$=\tfrac{1}{4}a^2\cot^{-1}\left(\frac{a^2-cd}{ac+ad}\right)-\tfrac{1}{4}b^2\cot^{-1}\left(\frac{b^2-cd}{bc+bd}\right)+\tfrac{1}{4}c^2\tan^{-1}\left(\frac{ac-bc}{ab+c^2}\right)$$

$$+\tfrac{1}{4}d^2\tan^{-1}\left(\frac{ad-bd}{ab+d^2}\right)-\tfrac{1}{4}(a-b)(c+d).$$

(*b*) Let $r=4a\cos\theta/\sin^2\theta$....(1), $r=4b\cos\theta/\sin^2\theta$....(2), $r=4c\sin\theta/\cos^2\theta$....(3), $r=4d\sin\theta/\cos^2\theta$....(4), be the equations to the parabolas; $a>b$, $c>d$.

Let $\tan^{-1}\sqrt[3]{(a/c)}=\theta'$.

$$A=8a^2\int_{\theta'}^{\frac{1}{2}\pi}\frac{\cos^2\theta}{\sin^4\theta}d\theta+8c^2\int_0^{\theta'}\frac{\sin^2\theta}{\cos^4\theta}d\theta=\tfrac{16}{3}ac.$$

Similarly, $B=\frac{16}{3}ad$, $C=\frac{26}{3}bc$, $D=\frac{16}{3}bd$.

Area $ABCD=A+B-C-D=\frac{16}{3}(a-b)(c+d)$.

113. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College Defiance, Ohio.

Deduce the Sylvestrian Reciprocant from $x^4+y^4=4x^2y^2$.

Solution by the PROPOSER.

The given equation may be written $(x^2-y^2)^2=2x^2y^2$.

$\therefore\ x^2-y^2=xy\sqrt{2}$....($a$).

Put $(y/x)=w$; then from (a), by dividing by xy, etc., we get $w-w^{-1}=-\sqrt{2}$.

$\therefore\ w=\frac{1}{2}(-\sqrt{2}\pm\sqrt{3}),=m$....($b$).

$\therefore\ y=mx$, and $dy/dx=m$....(c).

Eliminating m from (c) by a second differentiation, we have $d^2y/dx^2=0$....(d).

Adopting Professor Sylvester's notation for reciprocants, viz: $dy/dx=t$, $d^2y/dx^2=a\ 2!$, $d^2y/dx^3=b\ 3!$, etc., we obtain from (d) the *first* pure Sylvestrian Reciprocant. All reciprocants *not* containing $t=dy/dx$ are *pure*; all others are *mixed*. The first two pure reciprocants are a and $4ac-5b^2$, and the first two mixed ones are $(1+t^2)b-2a^2t$ and $bt-a^2$. See our paper on Reciprocants published in the MONTHLY of November, 1894.

Also solved by *G. B. M. ZERR*.

114. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

When the sun's declination was 15° N. his altitude was found to be 20°, and after an hour's interval his altitude was found to be 31°. Required, the latitude of the place of observation.

Solution by G. B. M. ZERR, A.M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia. Pa.

Let $a=20^\circ=$altitude, $a'=31^\circ=$altitude, $\varphi=$latitude, $\delta=15^\circ=$declination, $\theta=15^\circ=$sun's angular path for one hour, $h=$hour angle.

$$\therefore\ \cos h=\frac{\sin a-\sin\varphi\sin\delta}{\cos\varphi\cos\delta},\ \cos[h-\theta]=\frac{\sin a'-\sin\varphi\sin\delta}{\cos\varphi\cos\delta}.$$

Eliminating h, we get

$$[\sin a-\sin\varphi\sin\delta]\cos\theta+\{\cos^2\varphi\cos^2\delta-[\sin a-\sin\varphi\sin\delta]^2\}^{\frac{1}{2}}\sin\theta=\sin a'-\sin\varphi\sin\delta.$$

But $\theta=\delta$. $\therefore\ \sin^2\varphi\sin^2\delta[2+\cos\delta]-2\sin\varphi\sin\delta[1-\cos\delta][\sin a+\sin a']$

$$=\cos^2\delta\sin^2\delta+2\sin a\sin a'\cos\delta-\sin^2 a-\sin^2 a'.$$

$$\therefore\ .067055\sin^2\varphi-.015126\sin\varphi=.020553;\ \sin^2\varphi-.2256\sin\varphi=.3065.$$

$$\sin\varphi=.6778 \text{ or } -.4522;\ \varphi=42^\circ\ 40'\ 30''.$$

115. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College Defiance, Ohio.

Determine geometrically where to stand so as to be able to throw a stone over a tree with the minimum velocity.

Solution by G. B. M. ZERR, A.M.. Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

The velocity of projection is the same as a body would acquire in falling from the directrix of the parabolic path to the point of projection.

∴ The velocity will be a minimum when the directrix is the least distance above the top of the tree. This is the case when you stand at the base of the tree, then the directrix passes just above the tree.

PROBLEMS FOR SOLUTION.

ARITHMETIC.

164. Proposed by JOSEPH V. COLLINS, Ph. D., Professor of Mathematics, State Normal School. Stevens Point, Wis.

Three women, the first with ten eggs, the second with thirty, and the third with fifty, went to market. They each got the same for their eggs, and all returned with the same money. What did they get?